

## Test 2

1. Consider the relation  $R = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 3), (3, 4)\}$  on the set  $\{1, 2, 3, 4\}$ . Is  $R$  reflexive, irreflexive, symmetric, antisymmetric, total, and/or transitive?
2. Is the relation  $R = \{(1, 2), (3, 4)\}$  on the set  $\{1, 2, 3, 4\}$ , reflexive, irreflexive, symmetric, antisymmetric, total, and/or transitive?
3. Show that if a relation is total, then it is reflexive.
4. Give an example of a relation which is both reflexive and irreflexive.
5. Give an example of a relation which is both symmetric and antisymmetric.
6. Can a relation be symmetric and transitive but not reflexive?
7. Consider the relation on the set of all living people  $a R b$  if  $a$  is an ancestor of  $b$ . Is  $R$  reflexive, irreflexive, symmetric, antisymmetric, total, and/or transitive?  
What is  $R^{-1}$ , expressed in English?
8. Let  $R$  be a relation on  $A$ . Show that if  $R$  has any of the properties reflexive, irreflexive, symmetric, antisymmetric, total and/or transitive, then the inverse relation  $R^{-1}$  has exactly the same properties.
9. Let  $A = \{1, 2\}$  and  $B = \{3, 4\}$ . Write down all functions  $f : A \rightarrow B$ . Which are surjective and which are injective? Write down the inverse functions of any bijective functions.
10. Let  $A$  and  $B$  be finite sets with  $|A| < |B|$ . Can a function  $f : A \rightarrow B$  be surjective?