6. Equivalence Classes and Partitions

1. Let $E = \{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$ on $\{1,2,3,4\}$. Show that E is an equivalence relation, and list its equivalence classes.

2. In a previous homework problem it was shown that the equivalence relation defined by $a \equiv b$ if $2 \mid (a-b)$ was an equivalence relation. What are its equivalence classes?

3. Let $X = \{1,2,3,4\}$ and let $\mathcal{P} = \{\{1\},\{2,3,4\}\}$. Write out the relation $\overset{\mathcal{P}}{\equiv}$ as a set of ordered pairs.

4. Let $X = \{1,2,3,4,5\}$ and let $\sim$ be the equivalence relation $A \sim B$ if $|A| = |B|$ for A, $B \in \mathcal{P}(X)$. Find $[\{1,2,5\}]$.

5. Find all possible partitions of the set $\{1,2,3,4\}$.

6. Prove that if $\sim$ is an equivalence relation on X, then for any a, $b \in X$ we have $a \in [b]$ if and only if $b \in [a]$.

7. Prove that if R and S are two equivalence relations on a set X, then $R = S$ if and only if R and S have the same equivalence classes.

8. Let R be the equivalence relation defined on the set of all living humans by $a\,R\,b$ if a is a sibling of b. What is [you]?

9. Let $f : A \to B$ be a surjection. Show that

$$\mathcal{P} = \{\{a \in A : f(a) = b\} : b \in B\}$$

is a partition of A.