Sample Questions
Limits and Continuity

Question 1

In the space provided, sketch a graph which illustrates each of the following general situations:

(a) $\lim_{x\to c} f(x) \neq f(c)$
(b) $\lim_{x\to c} f(x) = \infty$
(c) $\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$

Question 2

Sketch the graph of a function $f(x)$ which satisfies the following conditions:

a. $\lim_{x\to 1} f(x) = 1$

b. $f(1) = 2$

c. $\lim_{x\to 2^-} f(x)$ does not exist

d. $f(x)$ is continuous on the interval $[-2, 1)$

Draw the graph clearly so that each of the above conditions is clearly illustrated. You do *not* need to provide a formula for the function.

Question 3

Write down the following limits:

(a) $\lim\limits_{x\to 3} 5x^2 - 3\tan(\pi x)$

(b) $\lim\limits_{\theta\to 0} \dfrac{\cos\theta - 1}{\theta}$

(c) $\lim\limits_{t\to 0} t\sin(1/t)$

Question 4

Write down the following limits:

(a) $\lim\limits_{x\to 3} 4x^2 - 3\sin(\pi x)$

(b) $\lim\limits_{t\to 0} \dfrac{t}{\sin t}$

(c) $\lim\limits_{h\to 0} \dfrac{\frac{1}{2+h} - \frac{1}{2}}{h}$

Question 5

Write down the following limits:

(a) $\lim\limits_{x\to 1} \dfrac{x}{x+1} + \cos(\pi x)$

(b) $\lim\limits_{h\to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}$

(c) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x} - 1}{\sqrt{x} + x + 3}$

Question 6

Write down the following limits:

(a) $\lim_{x\to 3} \dfrac{4x^2}{x+1} - \cos(\pi x)$

(b) $\lim_{t\to\infty} \dfrac{3t^2-5t+3}{7t^2+5t-1}$

(c) $\lim_{h\to 0} \dfrac{\sqrt{2+h}-\sqrt{2}}{h}$

Question 7

Write down the following limits:

(a) $\lim_{x\to 3} \dfrac{x^2-1}{3x+1} - 7\cos(\pi x)$

(b) $\lim_{\theta\to 0} \dfrac{\theta}{\sin\theta}$

(c) $\lim_{t\to\infty} \dfrac{5t^{3/2}-3t^2+1}{1-6t^{3/2}}$

Question 8

Do the following limits exist? If so, write down the value:

(a) $\lim_{x\to 1^+} \dfrac{|x-1|}{x-1}$

(b) $\lim_{x\to 1^-} \dfrac{|x-1|}{x-1}$

(c) $\lim_{x\to 1} \dfrac{|x-1|}{x-1}$

Question 9

Consider the function

$$f(x) = \begin{cases} \cos x & x \geq 0 \\ ax+b & x < 0 \end{cases}$$

For which values of b is $f(x)$ continuous at $x = 0$?

Question 10

Consider the function

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 12 & x = 0. \end{cases}$$

The limit

$$\lim_{x\to 0} f(x)$$

a. Does not exist, because $f(x)$ oscillates wildly.

b. Does not exist, because although the left-hand and right-hand limits exist, they are not equal.

c. Does exist and is equal to 0, because the Sandwich Theorem applies with $-x^2 \leq f(x) \leq x^2$.

d. Does exist and is equal to 12, because $f(0) = 12$.

e. None of the above.

Question 11

Consider the function

$$f(x) = \cos x - x$$

on the interval $[0, \pi]$.

Which of the following statements are false, and which are true:

a. $f(x)$ is continuous.

b. $f(x) = 0$ at some point in $[0, \pi]$.

Question 12

On which of the following interval(s) must the function $f(x) = 2 + x^2 - x^3$ have a root? (Use the Intermediate Value Theorem)

a. $[-1, 0]$

b. $[0, 1]$

c. $[1, 2]$

d. $[2, 3]$

e. None of the above.