

Sample Questions

Integration

Question 1

Find the value of the following integrals:

(a) $\int_0^\pi x \sin(1 - x^2) dx$

(b) $\int_0^e \frac{1}{x} dx$

(c) $\int \cos^2 \theta d\theta$

Question 2

Given

$$F(x) = \int_1^x t \sin t dt,$$

find $\frac{dF}{dx}$.

Question 3

Find the value of the following definite integrals:

(a) $\int_{-1}^1 2x \sin(1 - x^2) dx$

(b) $\int_0^1 \frac{10\sqrt{v}}{(1 + v^{3/2})^2} dv$

Question 4

Verify, using substitution, that for any function $f(x)$,

$$\int_a^b f(x + c) dx = \int_{a-c}^{b-c} f(x) dx.$$

Question 5

By calculating Riemann sums, show that

$$\int_a^b x^2 dx = \frac{1}{3}(b^3 - a^3).$$

Question 6

Write down the following indefinite integrals

(a) $\int \sin^2 x \, dx$

(b) $\int 4u^3 - u^{-2/3} + t \, du$

(c) $\int \frac{\theta \sin(\theta^2)}{\sqrt{\cos(\theta^2)}} \, d\theta$

Question 7

Write down the following definite Integrals

(a) $\int_{\pi/2}^{\pi} \sin^2 x \, dx$

(b) $\int_0^1 4u^3 - u^{-2/3} + t \, du$

(c) $\int_0^2 \sqrt{4-x^2} \, dx$

Question 8

A rectangular swimming pool is 30 ft wide and 50 ft long. The table below shows the depth of the water $h(x)$ at 10 ft intervals from one end of the pool to the other.

Position x ft	0	10	20	30	40	50
Depth $h(x)$ ft	4.0	4.7	5.5	6.5	8.0	7.0

- (a) Estimate the volume of the pool using left-endpoint values.
(b) Estimate the volume of the pool using right-endpoint values.

Question 9

Evaluate the integral

$$\int \frac{x}{\sqrt{2x+1}} \, dx$$

using substitution.

Question 10

Write down the following integrals:

(a) $\int 3x^2 - 6x + \frac{1}{2\sqrt{x}} \, dx$

(b) $\int_0^1 4u^3 - u^{-2/3} + t \, du$

(c) $\int_0^2 \sqrt{4-x^2} \, dx$

Question 11

(a) Let $f(x)$ be a function with the property that

$$f(x+c) = f(x)f(c) \tag{1}$$

for all x and c . Show that

$$f'(x+c) = f'(x)f(c).$$

(b) Use part (a) to explain why the derivative of any exponential function $f(x) = a^x$ (where $a > 0$) at the point $x = c$ is of the form

$$f'(c) = k_a a^c$$

for some constant k_a .

(c) Let e be the number so that the constant k_e from part (b) is 1. Show that $k_a = \log_e a$.

Question 12

Write down the following indefinite integrals

(a) $\int \cos^2 x \, dx$

(b) $\int 3t^2 + \sqrt{t} - \frac{1}{t} \, dt$

(c) $\int \frac{x}{\sqrt{x^2+1}} \, dx$

Question 13

Write down the following definite Integrals

(a) $\int_0^{\pi/2} \cos^2 x \, dx$

(b) $\int_1^4 3t^2 + \sqrt{t} - \frac{1}{t} \, dt$

(c) $\int_0^{\pi/2} \sin \theta \sqrt{1 + \cos \theta} \, d\theta$

Question 14

Write down the value of

$$\frac{d}{dt} \int_1^t \cos u^2 \, du$$