Sample Questions
Derivatives

Question 1

Write down the following derivatives:

(a) $\frac{d}{dx}(12x^4 - 3x^3 + 7x^2 - 3x + 4)$

(b) $\frac{d}{dx}(\sin 2x)$

(c) $\frac{d}{dx}(\sec x)$

Question 2

Write down the following derivatives:

(a) $\frac{d}{dx}(16x^4 + 12x^2 - 3x^{-1})$

(b) $\frac{d}{d\theta}(\tan\theta \operatorname{cosec}\theta)$

(c) $\frac{d}{dt}((2t^3 + 1)(7 - 3t))$

Question 3

Suppose that functions $f(x)$ and $g(x)$ and their first derivatives have the following values at 0 and 1.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	1/3
1	3	-4	-1/3	-8/3

Write down the first derivatives of the following combinations at the given vaules of x:

(a) $5f(x) + g(x),\ x = 1$

(b) $f(x)g(x),\ x = 0$

(c) $f(g(x)),\ x = 0$

Question 4

Consider the following table of values for $f(x)$, $g(x)$ and their derivatives:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	1	-2	3
1	-1	-1	5	π

The derivative with respect to x of $y = f(g(x))$ at $x = 1$ is:

(a) 5π

(b) 15

(c) 0

(d) -2π

(e) None of the above.

Question 5

Consider the function $f(x) = \sqrt{x+1}$. Using limit techniques, find the derivative of the function at $x = 1$.
Verify your answer using the rules of differentiation.
Find the tangent and normal to the graph of the function at this point.

Question 6

Consider the function

$$g(x) = \frac{x+1}{x-1}$$

(a) Use the limit definition to find the derivative of $g(x)$.
(b) Verify your answer using the quotient rule.
(c) For which x is the slope of the tangent to the graph of $g(x)$ equal to -1.

Question 7

Differentiate

$$y = \left(\frac{\sqrt{7x+5}\cos x}{3x^2+5x-2} \right)^{50}$$

with respect to x.

Question 8

Consider the function

$$f(x) = \begin{cases} x\sin(1/x) & x \neq 0 \\ 0 & x = 0. \end{cases}$$

(a) Use the rules of differentiation to find the derivative of $f(x)$ for $x \neq 0$.
(b) Use the limit definition of the derivative to show that $f(x)$ is *not* differentiable at $x = 0$.
(c) Use the sandwich theorem to show that $f(x)$ is continuous at $x = 0$.

Question 9

Find the derivative of

$$h(t) = \left(\frac{(t-1)^2}{t+1} \right)^{1/2}.$$

Question 10

Using limit techniques, show that if f and g are differentiable functions, then

$$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}.$$

Question 11

The upper half of a circle or radius 1 is given by the equation $y = \sqrt{1-x^2}$. Using limit techniques, find the derivative.
Find the equation of a tangent through a general point $(c, \sqrt{1-c^2})$ on the semicircle.
Find the tangent line to the semicircle which passes through the point $(\sqrt{2}, 0)$.

Question 12

The position of an object is given by the formula

$$s(t) = \frac{4t}{t^2 + 1}.$$

Find the formula of the velocity of the object.
Find the velocity at $t = 0$ and $t = 1$.

Question 13

The product rule says that the first derivative of $y = uv$ is given by

$$\frac{dy}{dx} = \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.$$

Use the product rule to find a formula for the *second* derivative of y.
Use this formula to find the second derivative of $y = (x^2 + 1)\cos x$.

Question 14

Find

$$\frac{d^{103}}{dx^{103}}\left(\cos(2x)\right).$$

Question 15

The curve

$$2(x^2 + y^2)^2 = 25(x^2 - y^2)$$

is called a *lemniscate*. Find the equation of the tangent to the curve at the point $(3, 1)$.

Question 16

Find the tangent to the hyperbola

$$25x^2 - 9y^2 = 16$$

at the point $(1, 1)$.

Question 17

Find the derivative of

$$y = x^{1/x}.$$

Question 18

Use logarithmic differentiation to find the derivative of

$$y = \frac{(x^3 + 10x - \ln x)^3\sqrt{\sin x - 2x}}{(\tan^{-1} x + 1)^{3/2}}.$$

Question 19

Find the derivative of

$$f(x) = \cosh(e^x(x+1)).$$

Question 20

Find the derivative of

$$x \ln x - x.$$

Question 21

Two students, Bruce and Sheila, were asked to implicitly differentiate

$$\frac{x^2}{y} + y = 3.$$

(a) Bruce worked the problem directly, and obtained the answer $\frac{2xy}{(x^2-y^2)}$. Check Bruce's working.

(b) Sheila realized that multiplying both sides of the equation by y to get $x^2 + y^2 = 3y$ made the differentiation easier. However the answer she got was $\frac{2x}{(3-2y)}$. Check her working.

(c) Did one of the students make a mistake, or are they both correct? If you think there was a mistake, explain what it was; if you think they are both correct, explain why the two answers are equal.